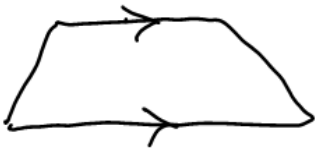
Trapezoids and Kites

VOCABULARY

Trapezoid

a quadrilateral w/ exactly one pair of parallel sides

Bases of a trapezoid



the pair of parallel sides

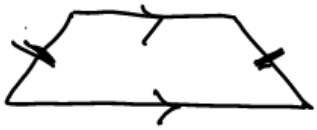
Base angles of a trapezoid

Either pair of angles whose common side is a base of a trapezoid

Legs of a trapezoid

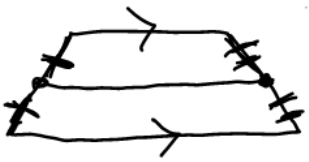
the nonparallel sides

Isosceles trapezoid



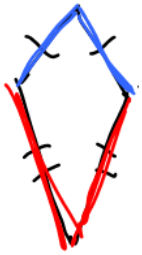
a trapezoid w/ congruent legs

Midsegment of a trapezoid



the segment that connects the midpoints of its legs

Kite



a quadrilateral that has two pairs of consecutive congruent sides, but opposite sides are not congruent.

THEOREM 8.14

If a trapezoid is isosceles, then each pair of base angles is congruent

$\angle A \cong \angle B, \angle C \cong \angle D$

THEOREM 8.15

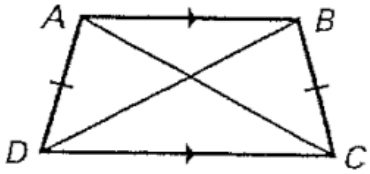
If a trapezoid has a pair of congruent base angles then it is an isosceles trapezoid.

ABCD is an isosceles trapezoid

THEOREM 8.16

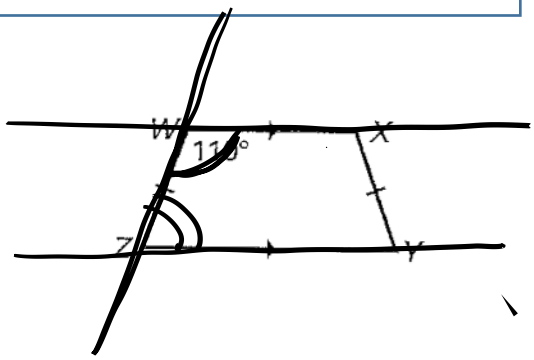
A trapezoid is isosceles if and only if its diagonals are congruent.

ABCD is isosceles if and only if $\overline{AC} \cong \overline{BD}$.



Example 1: ***Using Properties of Isosceles Trapezoids***

WXYZ is an isosceles trapezoid.
Find $m\angle X, m\angle Y, and\ m\angle Z$.



Solution

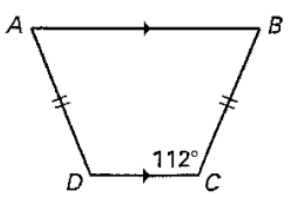
WXYZ is an isosceles trapezoid, so $m\angle X = m\angle W = 110°$.

$\angle W$ and $\angle Z$ are consecutive interior angles formed by parallel lines, so they are supplementary

$m\angle Z = 70° = m\angle Y$

√ **Find the angle measures of *ABCD*.**

7.



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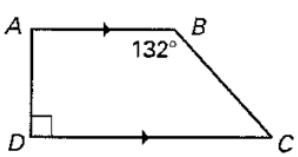

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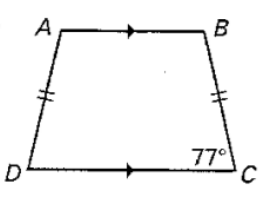
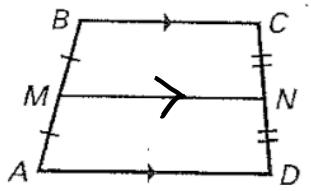


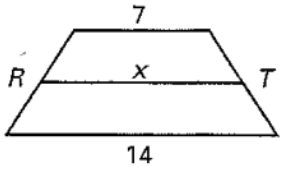
THEOREM 8.17: MIDSEGMENT THEOREM FOR TRAPEZOIDS

The midsegment of a trapezoid is parallel to each base and its length is one half the sum of the lengths of the bases.

$\overline{MN} \parallel \overline{BC}, \overline{MN} \parallel \overline{AD}, MN = \frac{1}{2}(BC + AD)$



√ Find the value of **x**.



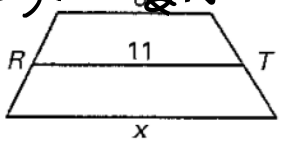
$x = \frac{1}{2}(7 + 14)$

$= \frac{1}{2}(21)$

$= 10.5$

$11 = \frac{1}{2}x + 4$

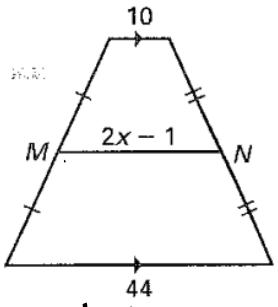
$(2)7 = (2)\frac{1}{2}x$



$(2)11 = \frac{1}{2}(x + 8)(2)$

$22 = x + 8$

$14 = x$



$2x - 1 = \frac{1}{2}(10 + 44)$

$2x - 1 = 27$

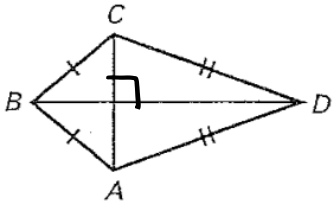
28

$x = 14$

THEOREM 8.18

If a quadrilateral is a kite, then its diagonals are perpendicular

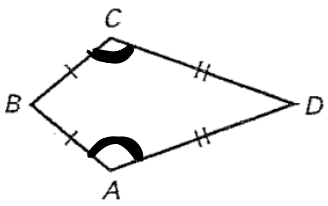
$\overline{CA} \perp \overline{BD}$



THEOREM 8.19

If a quadrilateral is a kite, then exactly one pair of oppposite angles are congruent.

If $\overline{BC} \cong \overline{BA}$, *then* $\angle A \cong \angle C$ *and* $\angle B \ncong \angle D$

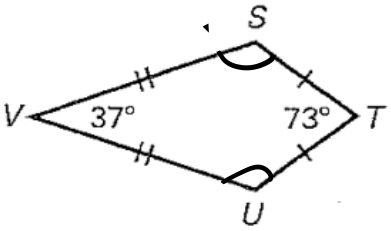


Example 2: ***Angles of a Kite***

Find $m\angle S$ *and* $m\angle U$.

Solution

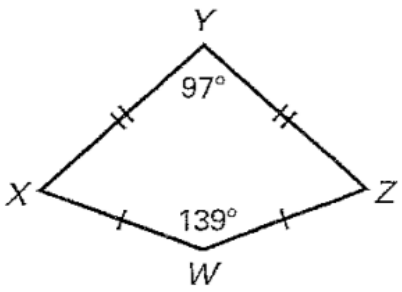
STUV is a kite, so $\angle S \cong \angle U$ and $m\angle S = m\angle U$



$37 + 73 = 110$

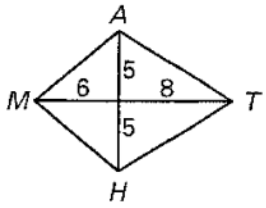
$360 - 110 = 250/2$ $m\angle S = m\angle U = 125°$

√ Find $m\angle X$ *and* $m\angle Z$.



Find the length of the sides to the nearest hundredth or the measure of the angles in kite *MATH*.

13.



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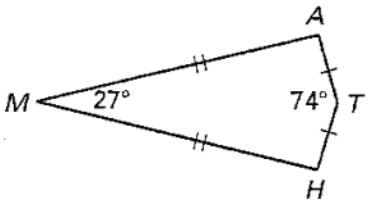

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